Simulation of Wave-Current Interaction Using Novel, Coupled Non-Phase and Phase Resolving Wave and Current Models

Andrew Kennedy
156 Fitzpatrick Hall
Department of Civil Engineering and Geological Sciences
University of Notre Dame
Notre Dame, IN 46556
phone: (574) 631-6686 fax: (574) 631-9236 email: andrew.kennedy@nd.edu

Award Number: N00014-11-1-0045
http://www.nd.edu/~akenned4/

LONG-TERM GOALS

The long term goals of this project are to be able to predict nearshore waves, currents, and sediment transport accurately from >20m water depth through to the shoreline. We would like to accomplish this over as large an area as possible; on the order of tens of km^2, and to resolve all individual waves. Time periods simulated would be of order hours to days at maximum. We would also like to be able to directly couple these phase-resolving models with non-phase resolving models for integration into larger scale dynamics.

OBJECTIVES

The specific objectives of this project, which began less than six months ago, are to (1) Develop and test novel, fundamentally rotational phase-resolving wave-current systems which may have arbitrary order; (2) Code these theoretical systems and develop them into phase-resolving nearshore surf zone models; and (3) Couple with large scale wave/circulation models.

For goal (1), more detailed objectives include combining aspects of Boussinesq and Green-Naghdi water wave theory to arrive at systems that retain the scaling and enhancement of properties through asymptotic rearrangement found in Boussinesq systems; and the fundamentally rotational behavior of Green-Naghdi systems. This will include linear and nonlinear properties such as dispersion, shoaling, and second harmonics. A second goal is to extend Boussinesq scaling to include dual irrotational-rotational scaling for arbitrary order. The system should have the general form of a Boussinesq-type system with mixed space-time derivatives.

For goal (2) detailed objectives are to achieve at a high combination of accuracy and efficiency; to develop a code that may be used with complex geometries; and one that will eventually be easy to parallelize. Our objective is also to make as few approximations as possible. We will produce both 2D and 3D codes.

For goal (3), our objective is to develop protocols to pass information back and forth with the large scale wave-circulation models, which will be SWAN/ADCIRC for the first development phase.

Report Documentation Page

Form Approved
OMB No. 0704-0188

1. REPORT DATE **30 SEP 2011**	2. REPORT TYPE	3. DATES COVERED **00-00-2011 to 00-00-2011**
4. TITLE AND SUBTITLE **Simulation of Wave-Current Interaction Using Novel, Coupled Non-Phase and Phase Resolving Wave and Current Models**		5a. CONTRACT NUMBER 5b. GRANT NUMBER 5c. PROGRAM ELEMENT NUMBER
6. AUTHOR(S)		5d. PROJECT NUMBER 5e. TASK NUMBER 5f. WORK UNIT NUMBER
7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES) **University of Notre Dame,Department of Civil Engineering and Geological Sciences,156 Fitzpatrick Hall,Notre Dame,IN,46556**		8. PERFORMING ORGANIZATION REPORT NUMBER
9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)		10. SPONSOR/MONITOR’S ACRONYM(S) 11. SPONSOR/MONITOR’S REPORT NUMBER(S)
12. DISTRIBUTION/AVAILABILITY STATEMENT **Approved for public release; distribution unlimited**		
13. SUPPLEMENTARY NOTES		
14. ABSTRACT		
15. SUBJECT TERMS		

16. SECURITY CLASSIFICATION OF:			17. LIMITATION OF ABSTRACT	18. NUMBER OF PAGES	19a. NAME OF RESPONSIBLE PERSON
a. REPORT **unclassified**	b. ABSTRACT **unclassified**	c. THIS PAGE **unclassified**	**Same as Report (SAR)**	**4**	

Standard Form 298 (Rev. 8-98)
Prescribed by ANSI Std Z39-18

APPROACH

All of the budget for this project goes to fund PhD student Yao Zhang, who is the major worker. Advisor and PI Dr. Andrew Kennedy is also working on this project.

Our fundamental technical approach is to represent nearshore water wave systems by retaining Boussinesq scaling assumptions, but without any assumption of irrotationality. We continue to assume a polynomial variation in horizontal velocity

$$\mathbf{u}(x, y, z; t) = \sum_{j=0}^{N} \tilde{\mathbf{U}}_n(x, y; t)\, f_n(q) \tag{1}$$

where $\boldsymbol{u}$ is the horizontal velocity, $f_n(q)$ is a polynomial function of $q \equiv (z+h)/(h+\eta)$, and $\tilde{\boldsymbol{U}}_n$ are coefficients that vary in horizontal coordinates and time. The specification of N, which controls the order of approximation, and f_n, which allows for asymptotic rearrangement, determines the system properties once the velocity expansion is integrated into Boussinesq-scaled continuity and Navier-Stokes equations. This is a generalization of the Boussinesq approach that allows for much more freedom in determining the system properties.

The resulting systems can have two forms: a classic Boussinesq-like appearance with mixed space-time derivatives but with several coupled equations; or a scaled pressure-Poisson-like form with polynomial vertical variation. Each has advantages for certain cases. We note that even though we are considering water wave systems, the scaled pressure-Poisson form may be quite useful for weakly nonhydrostatic ocean models.

A variant of these approaches uses dual irrotational-rotational scaling. This retains the weakly nonhydrostatic conditions but adds additional modes to examine, for example, depth-varying undertow, longshore current structure, and so on. This approach will be particularly useful when sediment transport studies are initiated.

WORK COMPLETED

To date, most of the work performed has been on deriving the theoretical systems and exploring their properties. We have developed scalings, and derived both general systems and variants for specific levels of approximation (e.g. $O(\mu^2)$, $O(\mu^4)$). We have also determined analytical properties for systems up to quite high levels of approximation, and optimized properties using asymptotic rearrangement. This rearrangement can improve properties greatly, and because of our very general approach we are able to simultaneously optimize both linear dispersion and shoaling. We note that we have also learned how to do this for standard Boussinesq-type equations, and this is something that we will be communicating to others. Two papers are in preparation.

We have begun numerical implementation in a Discontinuous Galerkin finite element framework, but this has not yet reached a stage where results may be shown.

RESULTS

Our most significant results to date have been in deriving the systems, determining their analytical properties up to high order, and in optimizing the systems to improve their properties without any resulting increase in computational expense. We have also determined ways to rearrange the systems so that parts may be solved sequentially and not simultaneously, which will further increase efficiency. Figure 1 shows linear dispersion relations (the most basic measure of system accuracy) for levels of approximation $O(\mu^2)$, $O(\mu^4)$, $O(\mu^6)$, and $O(\mu^8)$, compared to the exact linear dispersion relation. When simple polynomial $(z+h)^n$ basis functions are used, the top curves result. These show improved performance with increasing order, but not quickly. In contrast, the more optimized curves on the bottom show much faster convergence for the same level of approximation. We have shown analytically that for an $O(\mu^N)$ system, we can easily optimize dispersion to be at level $O(\mu^{2N-2})$, which is a huge improvement. Even higher accuracy is possible for dispersion, but at the cost of decreased accuracy in other properties, so we are not pursuing these systems.

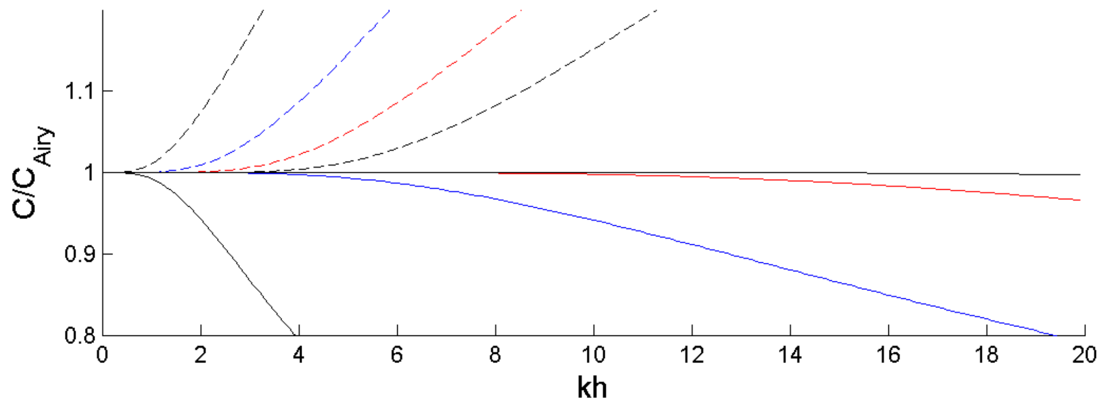

Figure 1. Simple polynomial (top) and optimized (bottom) dispersion relationships for the new systems for (left-right) orders 2, 4, 6, and 8. Because the optimized relationships lie much closer to 1, they are more accurate for the same level of approximation.

This increased accuracy follows through to other properties. Figure 2 shows that the linear shoaling gradient also has very good properties that improve with the order of approximation. Nonlinear properties (not shown) such as second harmonic also show good accuracy. One major benefit of these systems is that convergence appears to be uniform with increasing order, while standard Boussinesq systems diverge at higher order.

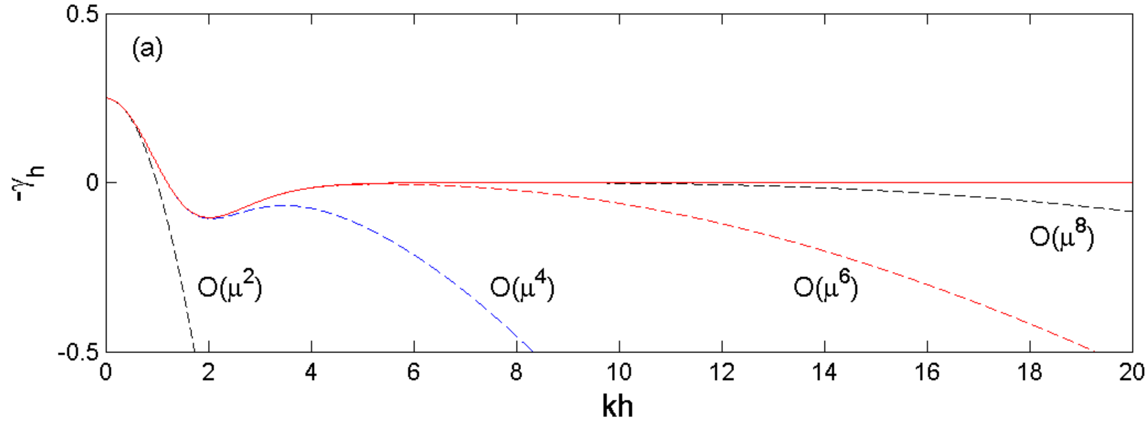

Figure 2. Linear shoaling gradient for optimized basis functions (dashed) compared to exact relation (solid line).

Another result that is still ongoing, is that the two different forms of the pressure formulation (mixed space-time/Pressure Poisson) appear to give identical results for compared properties. Although we are still confirming that this applies to all properties, it is a very encouraging result and may be used to confirm accuracy.

IMPACT/APPLICATIONS

Because of assumptions made in their derivation, existing Boussinesq models have in some ways reached hard limits. Because of this, their ability to simulate processes such as sediment transport, or the ease of extending to higher order have been dissapointing in some respects. The present formulations retain almost all of the Boussinesq ability to manipulate equations asymptotically while removing many of their limitations with respect to rotationality. One form of these new systems also seems to be very amenable to higher order formulation. When coding and testing is complete, the new systems should combine high accuracy in wave shape with the ability to simulate surf zone velocities and use these as forcing for sediment transport studies. The natural inclusion of vorticity should also improve predictions of nearshore currents and their evolution. These detailed capabilities presently do not exist and can be used to examine both storm impacts and typical nearshore processes.

RELATED PROJECTS

This project is directly tied to NSF project 1025519, which is a collaboration between Notre Dame, the University of Texas, and the Ohio State University. The present project has funding for a PhD student, Yao, Zhang, to work on these topics in collaboration with the other workers.